



# RELIABILITY EVALUATIONS OF 3-D FRAME SUBJECTED TO NON-STATIONARY EARTHQUAKE

A. CHAUDHURI

*Department of Civil Engineering, Indian Institute of Science, Bangalore 12, India*

AND

S. CHAKRABORTY

*Department of Civil Engineering, Bengal Engineering College (A Deemed University),  
Howrah 711103, West Bengal, India*

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The objective of the present work is to evaluate the integrated reliability of multistoried space frame subjected to random earthquake. The stochastic ground motion is described by fully non-stationary sigma-oscillatory model. The stochastic dynamic analysis is performed in the frequency domain to obtain the power spectral density function of random response. Finally, the reliability formulations are developed based on computed random response through the solution of first passage problem. A building frame idealized as a space frame in finite element modelling is considered for reliability analysis. Simple modal analysis is also performed for comparison of results.

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## 1. INTRODUCTION

Earthquake is a source of critical loading condition for all land-based structures located in seismically active region of earth. Due to its disastrous consequence, it is given more importance. Earthquake resistance analysis has been introduced for safe and reliable design and construction of many important earthquake-sensitive structures like buildings, bridges, dams, harbor, etc., need to be constructed in earthquake-prone area for the sake of development. In present earthquake resistance design practices, it is common to characterize the design ground motion in the form of a set of design spectra and to analyze the structure for the corresponding lateral loads. However, the response of all structures under the random ground excitation during earthquake is random in nature. Thus, a reliability-based design in the framework of random vibration analysis will provide a realistic and consistent basis for aseismic design of such structures.

Various models of ground motion due to earthquake are available in the existing literature such as white-noise model [1], Kanai–Tajimi stationary model [2], model using modulating function [3–6], random pulse train model [7], sigma-oscillatory process model [8, 9], etc. The computation of random responses considering these models are well documented in the theory of random vibration [10–12]. Though most of the literature consider the computation of extreme random response considering different ground motion models, the probabilities that these extreme responses have ever exceeded some

critical levels during the time interval are either neglected or tried [13–18] with simplified assumptions restricted to very simple structural models.

The basic objective of the present work is to present a reliability evaluation method utilizing the random response of structure subjected to non-stationary earthquake. In doing so random earthquake is modelled as sigma-oscillatory processes [9]. The random response of the structure is obtained in frequency domain. The expected rate of conditional up-crossing at a certain barrier level is used to determine the reliability of structure. The conditional up-crossing rates are obtained based on two-state Markov process [12, 19]. A building frame idealized as a space frame in finite element modelling is taken up to elucidate the proposed algorithm.

## 2. DYNAMIC EQUILIBRIUM EQUATION

The equation of motion for a multi-degree-of-freedom (m.d.o.f.) system under ground acceleration can be readily expressed as

$$[m]\{\ddot{u}(t)\} + [c]\{\dot{u}(t)\} + [k]\{u(t)\} = -[m][L]\{\ddot{u}_g(t)\}, \quad (1)$$

where  $[m]$ ,  $[c]$  and  $[k]$  are the global mass, damping and stiffness matrixes of the structure respectively. The mass and stiffness matrices for each element are derived by standard finite element method using two noded space frame elements having six degrees of freedom (d.o.f.) at each node.  $\{u(t)\}$  is the total displacement vector of the structure and  $\{\ddot{u}_g(t)\}$  is ground acceleration vector comprising six components (three translational and three rotational) which do not vary spatially.  $[L]$  is the influence coefficient matrix and  $j$ th column of  $[L]$  represents the pseudo-elastic response in all degrees of freedom due to unit support motion in the  $j$ th direction. In general, the rotational components of ground motion are considered to be negligible assuming that the ground is very stiff in shear and vertical component of ground motion is not considered in the present study. The ground motion that has only one horizontal component may act along any direction with respect to the principal axis of the structure depending on the location of the epicenter with respect to the structure. The ground motion is resolved into two components along the principal axis of the structure. Thus equation (1) becomes

$$[m]\{\ddot{u}(t)\} + [c]\{\dot{u}(t)\} + [k]\{u(t)\} = -[m][L] \begin{Bmatrix} r_1 \\ r_2 \end{Bmatrix} \ddot{u}_g(t), \quad (2)$$

where  $r_1$  and  $r_2$  are the direction cosines of the ground motion.

In the present numerical study for ease of analysis the damping matrix is formed by Rayleigh or proportional method, i.e.,

$$[c] = [m] \sum_{i=0}^n a_i [[m]^{-1} [k]]^i, \quad (3)$$

where  $a_i$  ( $i = 1, 2, 3, \dots, n$ ) are determined from the modal damping coefficient. The first two terms are taken into account considering 10% damping for vibration in the first two modes. For simplicity of analysis and ease of comparison proportional damping is considered here. However, it is to be mentioned that in the proposed complex inverse method the damping matrix need not to be restricted in the above form and it can be taken in the form of general proportional damping following Caughey [20].

### 3. FREQUENCY-DOMAIN ANALYSIS

#### 3.1. COMPLEX INVERSE

The analysis is performed in frequency domain. To transfer the dynamic equilibrium equation in frequency domain,  $\{U(\omega)\}$  and  $\ddot{U}_g(\omega)$  i.e., the Fourier transform of displacement vector  $\{u(t)\}$  and ground acceleration  $\ddot{u}_g(t)$ , respectively, are used. The Fourier transform of displacement and ground acceleration can be readily written as

$$\{U(\omega)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \{u(t)\} \exp(-i\omega t) dt, \quad \ddot{U}_g(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \ddot{u}_g(t) \exp(-i\omega t) dt. \quad (4a, b)$$

Substituting equations (4a) and (4b) in equation (2) the equilibrium equation transforms to

$$[-\omega^2[m] + i\omega[c] + [k]]\{U(\omega)\} = -[m][L] \begin{Bmatrix} r_1 \\ r_2 \end{Bmatrix} \ddot{U}_g(\omega),$$

i.e.,

$$[D(\omega)]\{U(\omega)\} = -[m][L] \begin{Bmatrix} r_1 \\ r_2 \end{Bmatrix} \ddot{U}_g(\omega), \quad (5)$$

where  $[D(\omega)]$  is the dynamic stiffness matrix complex in nature and function of frequency. Equation (5) can be solved directly by complex inversion as follows:

$$\{U(\omega)\} = -[D(\omega)]^{-1}[m][L] \begin{Bmatrix} r_1 \\ r_2 \end{Bmatrix} \ddot{U}_g(\omega) = \{H(\omega)\} \ddot{U}_g(\omega), \quad (6)$$

where  $\{H(\omega)\}$  is the complex frequency response function. The solution procedure is the same as static problem but all computations are to be performed in complex domain for different frequencies.

#### 3.2. MODAL DECOMPOSITION

For comparison of results, simple modal decomposition method is also performed. In modal decomposition method, the displacement  $\{u(t)\}$  is expressed as

$$\{u(t)\} = \sum_{i=1}^N \{\phi\}_i y_i(t), \quad (7)$$

where  $\{\phi_i\}$  and  $y_i(t)$  are the mode shape and generalized response of  $i$ th mode respectively. Substitution of  $\{u(t)\}$  in equation (2) yields a set of equations of motion for s.d.o.f. systems. Equation of motion for  $i$ th mode of vibration can be expressed as

$$M_i \ddot{y}_i(t) + C_i \dot{y}_i(t) + K_i y_i(t) = -\{\phi_i\}^T [m][L] \begin{Bmatrix} r_1 \\ r_2 \end{Bmatrix} \ddot{u}_g(t), \quad (8)$$

where  $K_i$ ,  $M_i$  and  $C_i$  are the usual modal stiffness, mass and damping of  $i$ th mode of vibration respectively. If  $Y_i(\omega)$  is the Fourier transform of response  $y_i(t)$ , the frequency domain form of the time-domain equation in the (8) transfers to

$$(K_i - \omega^2 M_i + i\omega C_i) Y_i(\omega) = -\{\phi_i\}^T [m][L] \begin{Bmatrix} r_1 \\ r_2 \end{Bmatrix} \ddot{U}_g(\omega),$$

i.e.,

$$Y_i(\omega) = -\frac{1}{(K_i - \omega^2 M_i + i\omega C_i)} \{\phi_i\}^T [m][L] \begin{Bmatrix} r_1 \\ r_2 \end{Bmatrix} \ddot{U}_g(\omega) = \tilde{H}_i(\omega) \ddot{U}_g(\omega). \tag{9}$$

Here,  $\tilde{H}_i(\omega)$  is the complex frequency response function corresponding to  $i$ th natural mode. Once  $Y_i(\omega)$  are obtained for different modes, the total  $\{U(\omega)\}$  considering ‘ $m$ ’ number of modes can be obtained as

$$\{U(\omega)\} = \sum_{i=1}^m \{\phi_i\} Y_i(\omega) = \sum_{i=1}^m \{\phi_i\} \tilde{H}_i(\omega) \ddot{U}_g(\omega) = \{\tilde{H}(\omega)\} \ddot{U}_g(\omega). \tag{10}$$

#### 4. GROUND MOTION MODEL

The ground motion due to earthquake is characterized by a sudden rise and slow decay i.e., mean and auto-correlation function or power spectral density function (PSDF) are not time invariant. But the well-established Kanai–Tajimi model does not show any time dependency of the PSDF of ground motion. Though the motion becomes stationary after some time and can be suitably modelled by Kanai–Tajimi model, sudden rise of ground acceleration may become critical for satisfactory performance and safety of various structures. To incorporate the fully non-stationary character of ground motion, it is described as a sigma-oscillatory process as suggested by Conte and Peng [9]. In this model, the oscillatory processes are pair-wise independent. The ground acceleration is expressed as

$$\ddot{u}_g(t) = \sum_{k=1}^p X_k(t) = \sum_{k=1}^p A_k(t) B_k(t), \tag{11}$$

where  $p$  is the number of component processes  $X(t)$  and  $A_k(t)$  is the time-dependent modulating function expressed as modified gamma function:

$$A_k(t) = \alpha_k (t - \zeta_k)^{\beta_k} \exp(-\gamma_k (t - \zeta_k)) H(t - \zeta_k), \tag{12}$$

where  $\alpha_k$  and  $\gamma_k$  are positive constants,  $\beta_k$  is a positive integer,  $\zeta_k$  is the arrival time of  $k$ th component process  $X_k(t)$  and  $H(t)$  is a unit step function. The  $k$ th zero mean stationary Gaussian process  $B_k(t)$  is characterized by its power spectral density function

$$S_{B_k B_k}(\omega) = \frac{v_k}{2\pi} \left[ \frac{1}{v_k^2 + (\omega + \eta_k)^2} + \frac{1}{v_k^2 + (\omega - \eta_k)^2} \right], \tag{13}$$

where  $v_k$  and  $\eta_k$  are two parameters representing the frequency bandwidth and predominant or central frequency of the process  $B_k(t)$ . The corresponding evolutionary (time-varying) PSD function of ground acceleration is

$$S_g(\omega, t) = \sum_{k=1}^p |A_k(t)|^2 S_{B_k B_k}(\omega). \tag{14}$$

The parameters used in equations (12) and (13) are normally estimated such that the PSD function fits the best (in the least-squares sense) to the PSD function of the target earthquake accelerogram estimated using short-time Thomson’s multiple window method. In the present numerical study, the six parameters for each component process are used as provided in reference [9] corresponding to the EI Centro (1940) earthquake record. The PSD function and parameters are furnished in Figure 1 and Table 1.

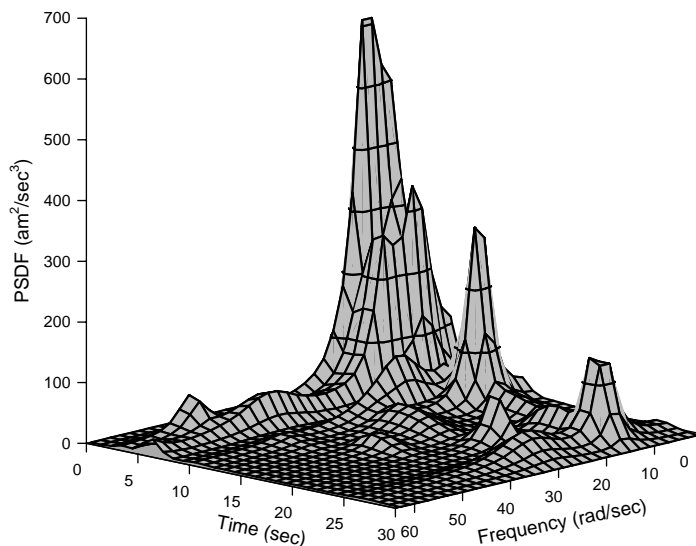


Figure 1. PSDF of ground acceleration due to 1940 EI Centro earthquake.

TABLE 1

*Estimated parameters of ground acceleration model for EI Centro 1940 Earthquake Record*

$K$	$\alpha_K$	$\beta_K$	$\gamma_K$	$\zeta_K$	$\nu_K$	$\eta_K$
1	37.2434	8	2.7283	-0.5918	1.4553	6.7603
2	104.0241	8	2.9549	-0.9857	2.4877	11.0857
3	31.9989	8	2.6272	1.7543	3.3024	7.3688
4	43.8375	9	3.1961	1.686	2.1968	13.5917
5	33.1958	9	3.1763	-0.0781	3.1241	14.3825
6	41.3111	9	3.1214	-7.096	6.7335	25.1532
7	4.2234	10	2.9904	-0.9464	2.6905	48.0617
8	19.9802	6	1.895	1.402	7.2086	37.6163
9	2.4884	10	2.6766	5.3123	6.1101	19.4612
10	24.1474	10	3.3493	8.8564	1.9862	9.04
11	2.5916	2	0.224	3.2558	2.4201	9.3381
12	2.2733	3	0.5285	16.2065	1.5244	14.1067
13	24.2732	3	1.0361	17.5331	1.7141	24.0444
14	41.3111	9	3.1214	-7.096	6.7335	25.1532
15	1.3697	10	2.5936	21.683	1.9362	12.9198
16	15.4646	2	0.7044	27.2979	1.7897	12.0205
17	0.0174	10	1.8451	-2.4168	4.9373	98.628
18	2.9646	10	3.1137	1.5751	1.9726	61.8316
19	0.0007	10	1.3686	2.5173	3.2479	43.90675
20	0.8092	4	0.5969	6.4396	3.6749	26.3365
21	16.7115	2	0.7294	12.493	1.7075	37.1139

## 5. DETERMINATION OF PSD FUNCTION

For linear systems with known complex frequency response function, the PSD function  $[S_u(\omega)]$  for displacements at any d.o.f. can be readily written as

$$[S_u(\omega)] = \{H(\omega)\}S_g(\omega)\{H^*(\omega)\}^T, \quad (15)$$

where  $\{H^*(\omega)\}$  is the complex conjugate of  $\{H(\omega)\}$ . For non-stationary ground motion, the PSD function of response will be

$$[S_u(\omega, t)] = \{H(\omega)\}S_g(\omega, t)\{H^*(\omega)\}^T. \quad (16)$$

The PSD function of ground motion  $S_g(\omega, t)$  as presented in earlier section is taken as input PSD function. The  $n$ th spectral moment of PSD about origin is

$$\lambda_n(t) = \int_{-\alpha}^{+\alpha} |\omega|^n S_i(\omega, t) d\omega, \quad (17)$$

where  $S_i(\omega, t)$  is the  $i$ th diagonal of matrix  $[S_u(\omega, t)]$  describing the PSD function of  $i$ th displacement degree of freedom. The mean square or variance of the response can be evaluated by putting  $\tau = 0$  in auto-correlation function or integrating PSD function with respect to frequency, i.e.,

$$\sigma_{u_i}^2(t) = \int_{-\infty}^{\infty} S_{u_i}(\omega, t) d\omega. \quad (18a)$$

In general, the displacements at various nodal d.o.f.'s are considered as the primary responses of the structure. The mean square velocity and acceleration of  $i$ th d.o.f. can be obtained as

$$\sigma_{\dot{u}_i}^2(t) = \int_{-\infty}^{\infty} S_{u_i}(\omega, t) d\omega = \int_{-\infty}^{\infty} \omega^2 S_{u_i}(\omega, t) d\omega, \quad (18b)$$

$$\sigma_{\ddot{u}_i}^2(t) = \int_{-\infty}^{\infty} S_{u_i}(\omega, t) d\omega = \int_{-\infty}^{\infty} \omega^4 S_{u_i}(\omega, t) d\omega. \quad (18c)$$

The PSD function for different responses is calculated at different frequencies  $\omega = n\varpi$  ( $\varpi$  is the fixed interval). In the present numerical study the integration is performed by conventionally used Simpson's one-third rule upto cut-off frequency, using the values of PSD function at discrete frequencies. However, the possibility of analytical solutions can be explored in future following the recent works of Conte and Peng [21, 22].

In the above, various statistics for displacements at different d.o.f.'s are derived. But for most of the cases the forces (axial forces, shears forces moments and torsion) at different d.o.f.'s of different elements are needed for the design. The force vector  $\{f(t)\}_j$  for  $j$ th element in local co-ordinate system can be written as

$$\{f(t)\}_j = [k]_j \{u(t)\}_j, \quad (19)$$

where  $[k]_j$  is the stiffness matrix for the  $j$ th element. In frequency domain the above equation for  $j$ th element becomes as

$$\{F(\omega)\}_j = [k]_j \{U(\omega)\}_j, \quad (20)$$

where  $\{F(\omega)\}_j$  is the Fourier transform of  $\{f(t)\}_j$ . Then, the PSD function of the force vector for  $j$ th element is

$$[S_f(\omega, t)]_j = [k]_j [S_u(\omega, t)]_j [k]_j^T. \quad (21)$$

Thus, the PSD function for  $i$ th force quantity of  $j$ th element  $f(t)_{ij}$  and its derivative  $\dot{f}(t)_{ij}$  can be easily obtained. The RMS value of  $i$ th force quantity of  $j$ th element and its derivative  $\sigma_{f_{ij}}(t)$  and  $\sigma_{\dot{f}_{ij}}(t)$  can be obtained similarly as done for displacement in equation (18a).

## 6. RELIABILITY ANALYSIS

The earthquake excitation is assumed to follow Gaussian distribution and as the structural behavior is linear, the responses will also be Gaussian. For any Gaussian random process the probability density functions (PDF) are obtained using the second-order statistics (i.e., r.m.s. values). For reliability analysis, the first passage failure criterion is considered. The expected up-crossing rate of a certain barrier level is used for the determination of the probability of no crossing. The reliability is defined as the probability that the absolute value of the response process will not exceed a specified threshold level from time  $t = 0$  to  $t$ . Thus, the probability of survival or reliability at any time is the same as the probability of no crossing. The reliability of the structure based on the first passage failure criterion for double-barrier problem can be obtained as [12]

$$R(\bar{u}, t) = R(\bar{u}, 0) \exp\left(-\int_0^t 2\alpha(\bar{u}, s) ds\right), \quad (22)$$

where  $\bar{u}$  is the barrier level and  $\alpha(\bar{u}, t)$  is the rate of crossing with positive slope of level  $u(t) = \bar{u}$  at time ' $t$ '. Note that  $R(\bar{u}, 0)$  is the reliability at time  $t = 0$  and for non-stationary case it can be assumed as unity.  $\alpha(\bar{u}, t)$  can be defined by the joint PDF of the response  $u(t)$  and its derivative  $\dot{u}(t)$ [12],

$$\alpha(\bar{u}, t) = \int_0^\infty \dot{u} p_{u, \dot{u}}(\bar{u}, \dot{u}, t) d\dot{u}. \quad (23)$$

In the present analysis, a simple approximation is made by assuming the joint PDF of  $u(t)$  and  $\dot{u}(t)$  as independent Gaussian process. This leads to the following simplified form of the PDF:

$$p_{u, \dot{u}}(u, \dot{u}, t) = \frac{1}{2\pi\sigma_u(t)\sigma_{\dot{u}}(t)} \exp\left(-\frac{u^2}{2\sigma_u^2(t)} - \frac{\dot{u}^2}{2\sigma_{\dot{u}}^2(t)}\right). \quad (24)$$

It is to be mentioned here that the PDF given above has a form which is correct only for stationary process. The true joint probability density of  $u(t)$  and  $\dot{u}(t)$  for a non-stationary process will contain the correlation between the two. This correlation may be small, but it cannot be ignored without placing limitations on the results. The assumptions made in the present work leads to considerable simplifications and it needs further investigations for the accuracy and robustness of the approximation.

Now, substituting equation (24) into equation (23), it becomes

$$\alpha(\bar{u}, t) = \frac{\sigma_{\dot{u}}(t)}{2\pi\sigma_u(t)} \exp\left[-\frac{1}{2}\left(\frac{\bar{u}}{\sigma_u(t)}\right)^2\right]. \quad (25)$$

When the value of the barrier level is large and the response is a wide band process the up-crossings are independent. In this situation reliability analysis using unconditional crossing rate  $\alpha(\bar{u}, t)$  is a good approximation based on the Poisson assumption that times between two up-crossings are independent. But for narrowband process with slowly varying amplitude  $A(t)$  and when the barrier level is not so high, a single up-crossing by  $A(t)$  is associated with several (almost uniformly spaced i.e., in clump) up-crossings by  $u(t)$ . This is inconsistent with the Poisson assumption. In this situation reliability is evaluated by a conditional crossing rate  $\eta(\bar{u}, t)$  rather than the unconditional crossing rate  $\alpha(\bar{u}, t)$ . Using Vanmarcke's modification based on two-state Markov process [19, 12], the conditional up-crossing rate is determined. For non-stationary process the crossing rate is

determined using time-dependent r.m.s. values:

$$\eta(\bar{u}, t) = \frac{\sigma_{\dot{u}}(t)}{2\pi\sigma_u(t)} \frac{1 - \exp(-\delta\sqrt{\pi/2}\bar{u}/\sigma_u(t))}{\exp(-\bar{u}^2/2\sigma^2(t)) - 1}, \tag{26}$$

where  $\delta = 1 - \lambda_1^2/\lambda_0\lambda_2$ . Finally, the reliability can be evaluated as

$$R(\bar{u}, t) = R(\bar{u}, 0)\exp(-2 \int_0^t \eta(\bar{u}, s) ds). \tag{27}$$

### 7. NUMERICAL EXAMPLE

An unsymmetrical 3-D building frame idealized as a space frame as shown in Figure 2 is subjected to the earthquake ground motion described by the PSDF as shown in Figure 1. The size of the columns and beams are 0.3 m × 0.3 m and 0.25 m × 0.45 m respectively. As most of the masses are concentrated at the roof level, the mass of the beam is taken 10 times that of the column. Mass density and modulus of elasticity of concrete members are taken as 2400 kg/m<sup>3</sup> and 2 × 10<sup>7</sup> kN/m<sup>2</sup> respectively.

The first three natural frequencies are computed as 7.75, 13.71 and 17.69 rad/s respectively. The results of modal analysis (truncated to three modes) are shown in the same figure with that of complex inversion for ease of comparison. The PSD function for displacement at node 16 and shear force at the base of column 2 along the X-axis due to ground motion along the X-axis are shown in Figures 3 and 4. Comparison of r.m.s. displacement at d.o.f. 91 (displacement along X-axis at node 16) are shown in Figure 5(a) and 5(b) and the r.m.s. value of shear force along the X-axis at the base of the column 2 are shown in Figure 6(a) and 6(b) due to ground motion along the X- and Y-axis respectively. Figure 7(a) and 7(b) represent the reliability of the structure with respect to displacement at d.o.f. 91 considering the barrier levels as 40 mm due to ground motion along the X- and Y-axis respectively. Figure 8(a) and 8(b) depicts the reliability of structure with respect to shear force of column 2 along the X-axis taking the barrier levels as 60 kN. The discrepancies of modal results from that of complex inversion is obvious as modal analysis results are truncated to three modes only.

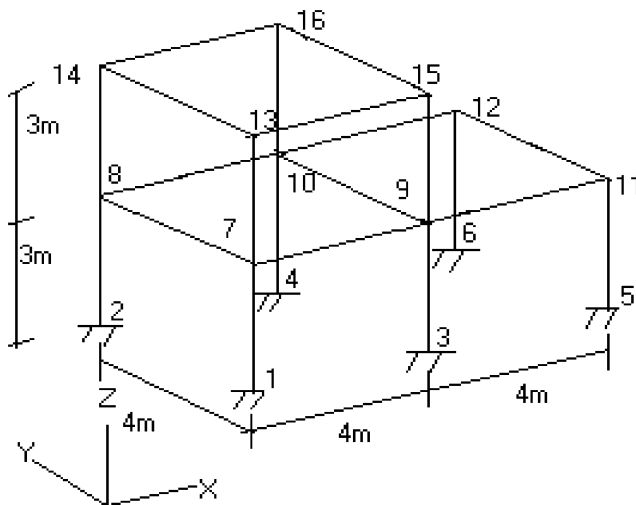


Figure 2. The two-storied unsymmetrical building frame.



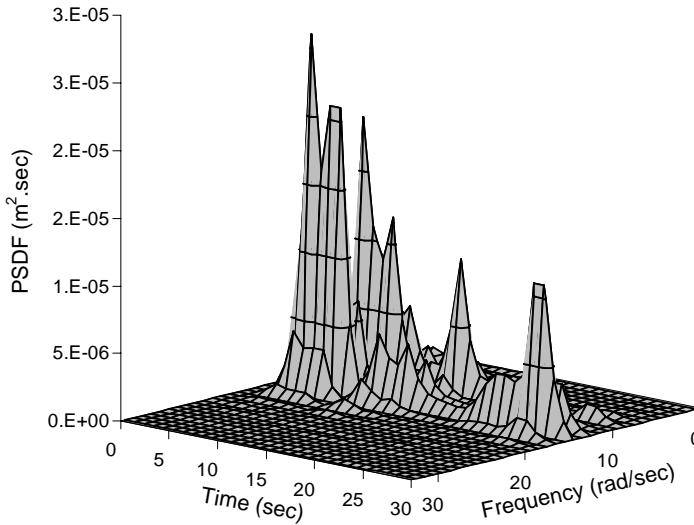


Figure 3. PSD function of displacement at node 16 along the  $X$  direction.

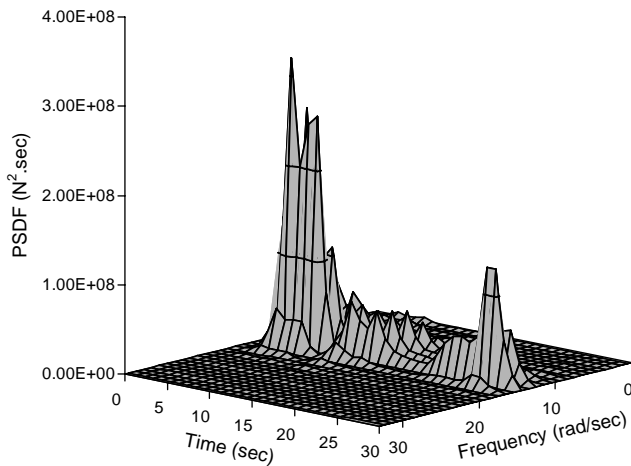


Figure 4. PSD function of shear force along the  $X$  direction at the base of column 2.

## 8. DISCUSSIONS AND CONCLUSIONS

The time-varying PSD function of ground motion (Figure 1) contains some peaks which correspond to the magnitude, arrival time as well as frequency content of the individual pulses. The PSD function of responses (i.e., displacement and shear) as shown in Figures 3 and 4 contain more prominent peaks at the natural frequencies of the structure in addition to the above-mentioned peaks. At natural frequencies the peaks are due to resonance phenomena. The response has some peak r.m.s. values just after the arrival of pulses. When r.m.s. value is high, the rate of crossing becomes high. After the arrival of strong pulses reliability falls. The reliability of the structure with respect to a response quantity depends on the barrier level specified for the safety of the structure. If the safety limit is

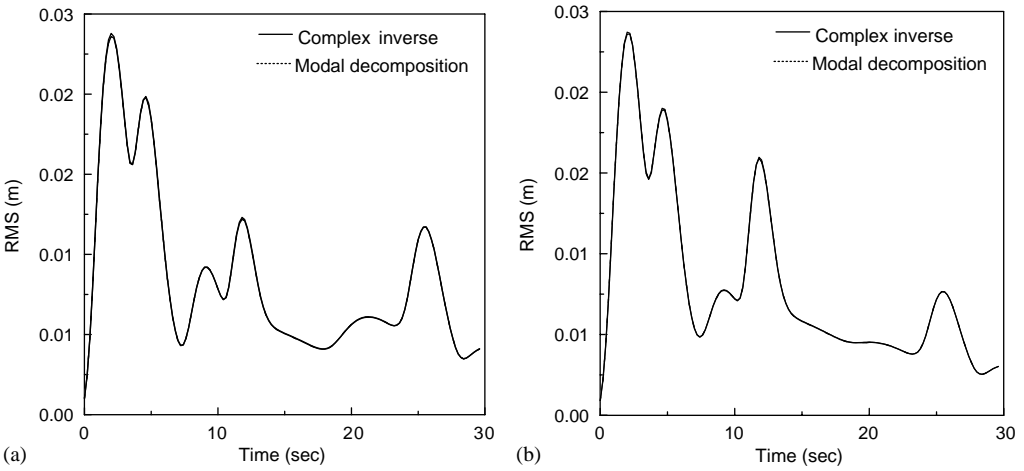


Figure 5. Comparison of r.m.s. value of displacement at node 16 along the X-axis due to the ground motion along: (a) X-axis (b) Y-axis.

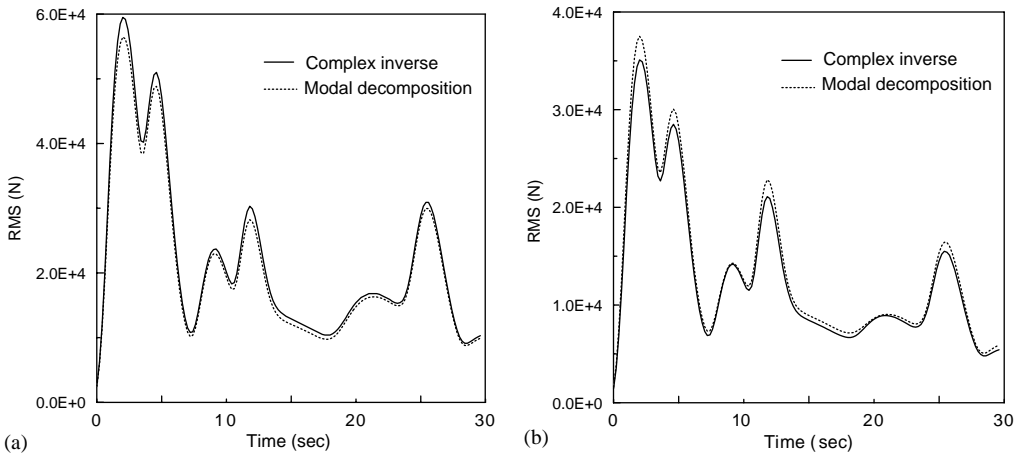


Figure 6. Comparison of r.m.s. value of shear force along the X-axis at the base of column 2 due to ground motion along: (a) X-axis, (b) Y-axis.

very high compared to the maximum r.m.s. value and duration of earthquake is less, the reliability will not be zero. But when the duration is long and frequent arrival of strong pulse is there, any barrier level cannot be expected safe. It is very difficult to predict the magnitudes of strong pulse, the number of pulses and their arrival rates. So any barrier level cannot be deemed safe however large it is. The above properties of the pulses associated with the ground motion can be expected from the knowledge of local soil condition and the location of site and previous earthquake records.

The r.m.s. values of responses are obtained by the complex inverse method as well as the modal decomposition method (taking the effect of first three modes). It is observed that the analysis in complex inverse method takes much time because the inversion of dynamic stiffness matrix is to be done for each frequency. For classically damped system modal

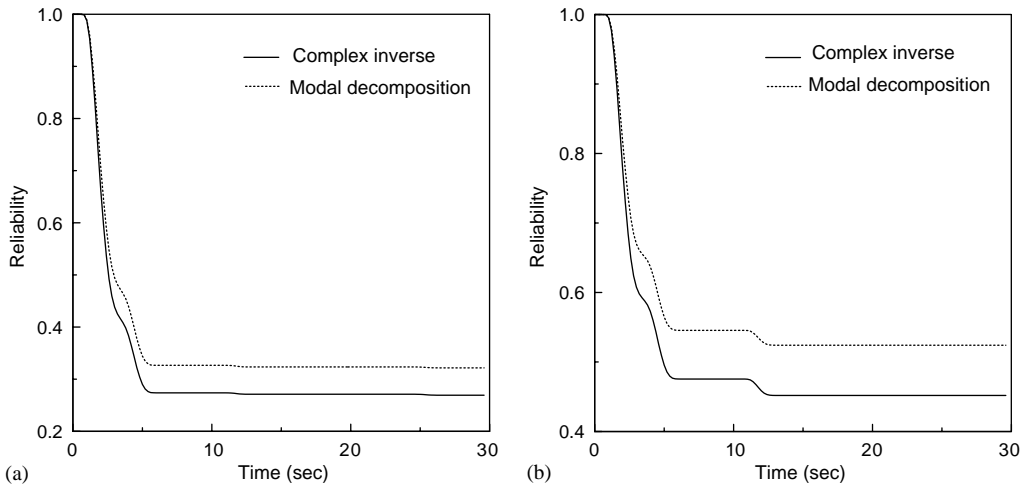


Figure 7. Comparison of reliability based on the displacement at node 16 along the  $X$ -axis due to ground motion along: (a)  $X$ -axis, (b)  $Y$ -axis.

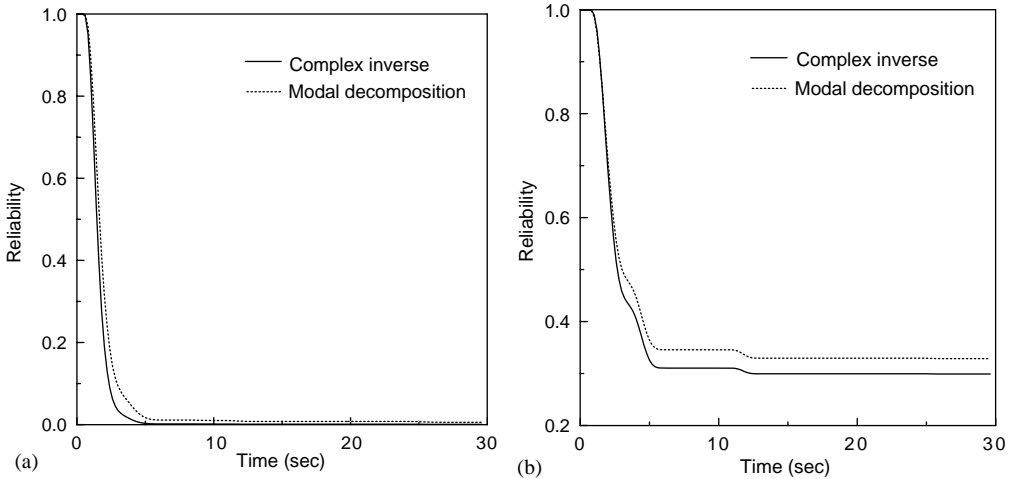


Figure 8. Comparison of reliability based on the shear force along the  $X$ -axis at the base of column 2 due to ground motion along: (a)  $X$ -axis, (b)  $Y$ -axis.

decomposition of the coupled differential equation is possible. In general, the effects of first few modes are significant. For each mode the analysis is the same as that of the s.d.o.f. system and time consumption is less. In the case of symmetric building where even distribution of stiffness and mass is possible for different stories, modal decomposition method using two or three modes give good results. But for unsymmetrical buildings with uneven distribution of stiffness and mass, it is well known that more number of modes are to be taken for modal analysis and the time of analysis becomes the same as the complex inverse method. In such cases complex inverse method is preferred as the effects of all modes are included.

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